CODIMENSION ONE FOLIATIONS ON $\mathbb{P}^3$ WITH CONIC SINGULARITIES

Joint work with D. Cerveau

A foliation of codimension one on a complex threefold M^3 has a generic conic singularity at m if after one blow-up $\pi\colon \widetilde{M^3} \to M^3$ at m the strict transform $\pi^{-1}\mathcal{F}$ is transversal to $\pi^{-1}(m)$ and defines a generic foliation on $\pi^{-1}(m) \simeq \mathbb{P}^2$.

The following conjecture was done by several authors:

Conjecture 1. *Let $\mathcal{F}$ be a codimension one holomorphic foliation on a complex manifold M of dimension ≥ 3. Then there are two possibilities:*

- *either $\mathcal{F}$ is the pull-back of some foliation on a complex surface,*
- *or $\mathcal{F}$ has some transverse projective structure outside a complex hypersurface $S \subset M$.*

On the other hand, it is known that a generic foliation of degree $d \geq 2$ on $\mathbb{P}^2$ has no transverse projective structure. This fact and Conjecture 1 motivate the following:

Conjecture 2. *If a codimension one foliation $\mathcal{F}$ on a complex manifold M of dimension three has a generic conic singularity, then $\mathcal{F}$ is a pull-back of a foliation on a surface.*

We solve Conjecture 2 in different situations when $M = \mathbb{P}^3$.